Epidemiology VHM 8120 Course
Winter 2023, Atlantic Veterinary College, PEI
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Lecture 2b: Linear Regression Diagnostics (VER 14.8-10)

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- <u>Learning objectives</u>
  - Assess assumptions linear regression
  - Identity outliers and influential observations
  - Develop approach linear reg. diagnostics
- <u>Exercises</u>
  - work on linear regression exercise 3
- <u>Quiz #2</u>

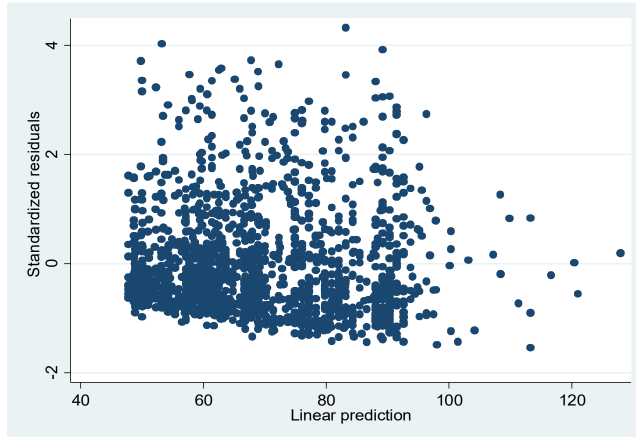
Example: WPC model (daisy2red.dta)

- VER example 14.12
  - outcome: wpc (interval from waiting period to conception)
  - predictors: parity, twin, dyst, rp, vag_disch, herd_size and herd_size2, calving_date (in months)
    - also interactions: i.rp##i.vag_disch
- Final (candidate) model

```
. reg wpc hs100_ctr hs100_ctr_sq parity1 i.aut_calv i.twin i.dyst i.rp##i.vag_disch

      Source |       SS           df       MS      Number of obs   =     1,574
-------------+----------------------------------   F(9, 1564)      =     13.22
       Model |  296062.694         9  32895.8549   Prob > F        =    0.0000
    Residual |  3892027.86     1,564  2488.50886   R-squared       =    0.0707
-------------+----------------------------------   Adj R-squared   =    0.0653
       Total |  4188090.56     1,573  2662.48605   Root MSE        =    49.885

------------------------------------------------------------------------------
         wpc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   hs100_ctr |   19.85708   2.163397     9.18   0.000     15.61361    24.10054
hs100_ctr_sq |   11.13827   3.111145     3.58   0.000     5.035817    17.24073
     parity1 |    1.13721   .8583103     1.32   0.185    -.5463501     2.82077
    aut_calv |
        yes  |  -8.263839   2.537751    -3.26   0.001    -13.24159   -3.286086
        twin |
        yes  |   20.68314   9.845165     2.10   0.036      1.37203    39.99425
        dyst |
        yes  |   11.70041   5.462576     2.14   0.032      .985666    22.41516
          rp |
        yes  |    5.98687   4.811976     1.24   0.214    -3.451734    15.42547
   vag_disch |
        yes  |   1.228196   7.161395     0.17   0.864    -12.81875    15.27514
rp#vag_disch |
    yes#yes  |   22.85194   12.51605     1.83   0.068    -1.698056    47.40194
       _cons |   64.33029   2.634114    24.42   0.000     59.16352    69.49705
------------------------------------------------------------------------------
```

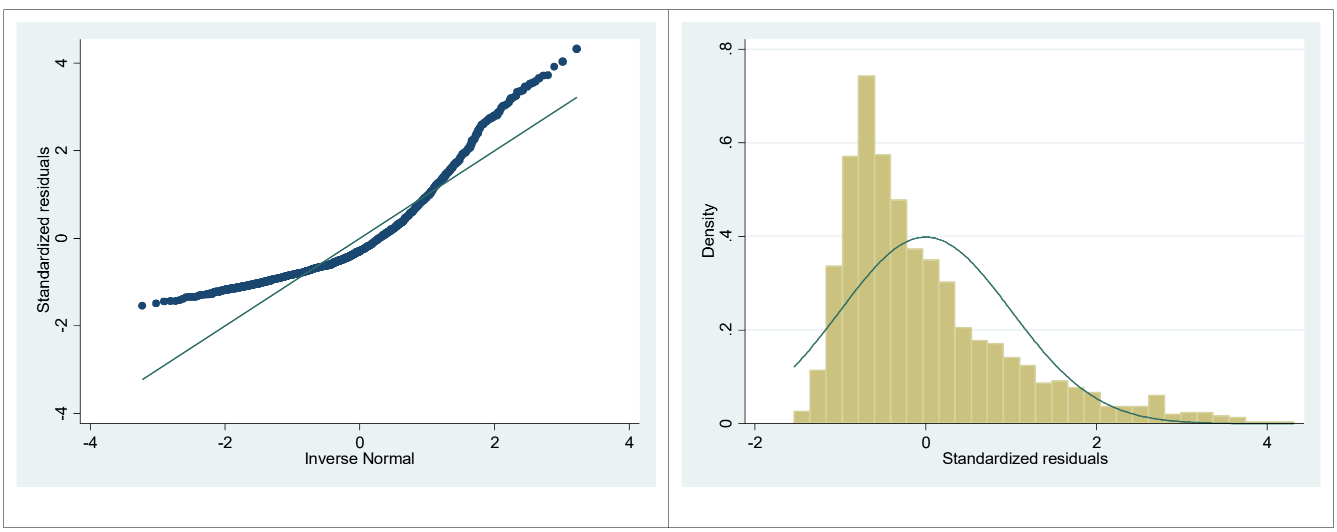
Evaluating major assumptions

- Homoscedasticity




```
. hettest

Breusch-Pagan / Cook-Weisberg test
for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of wpc

         chi2(1)      =    20.58
         Prob > chi2  =   0.0000

. imtest

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      74.11     44    0.0030
            Skewness |     143.84      9    0.0000
            Kurtosis |      33.27      1    0.0000
---------------------+-----------------------------
               Total |     251.22     54    0.0000
---------------------------------------------------
```

- Normality




```
. swilk  stdres

                   Shapiro-Wilk W test for normal data

    Variable |    Obs         W           V          z       Prob>z
-------------+--------------------------------------------------
      stdres |   1574     0.87871     115.660     11.977    0.00000
```

Transformation of Y (L1a)

- Box-cox
- Interpretation

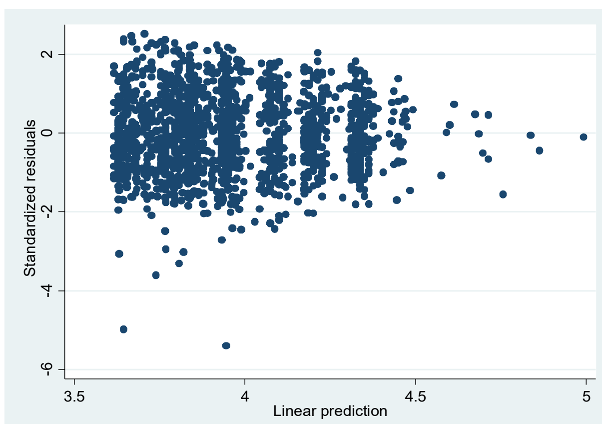
Re-assessing assumptions

- Log-transformed model

```
xi:boxcox wpc hs100_ctr hs100_ctr_sq parity1 i.aut_calv i.twin i.dyst
i.rp*i.vag_disch

...output omitted...

                                                  Number of obs   =       1574
                                                  LR chi2(8)      =     148.83
Log likelihood = -7960.5368                       Prob > chi2     =      0.000

------------------------------------------------------------------------------
         wpc |      Coef.   Std. Err.       z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /theta |   .1099104   .0271003     4.06   0.000     .0567947    .1630261
```

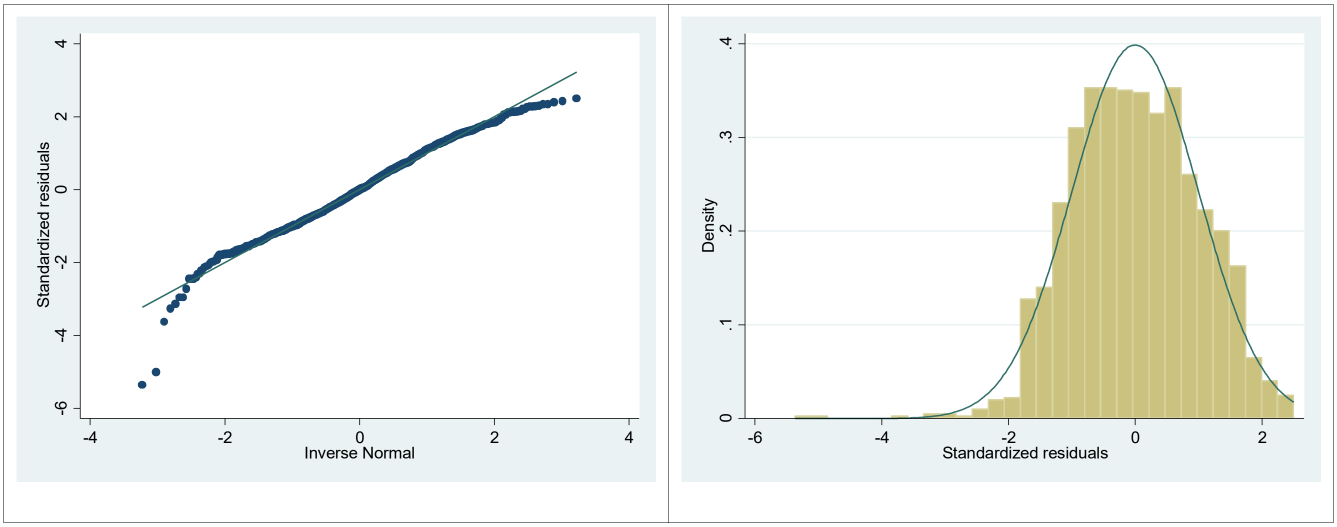
  - ☆ value of λ close to 0 -> log transformation

- Homoscedasticity (L1a: p11)



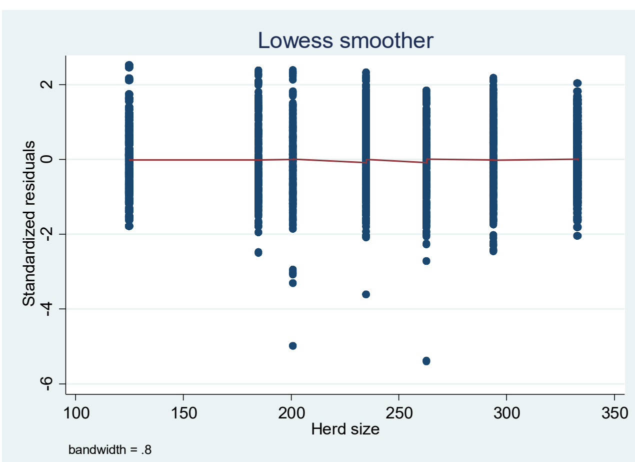
  - ☆ tests provide contradicting results
    - imtest – constant variance
    - however BPCW test highly significant

- Normality (L1a)

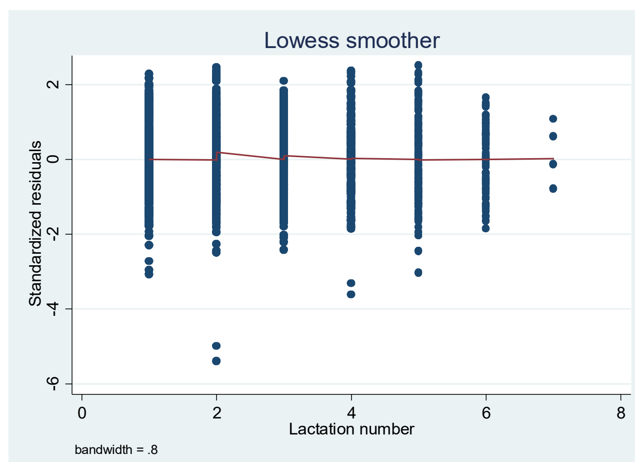


- Linearity (L1a)
  - continuous predictors only
  - standardized residuals vs predictor

Herd size



Parity



  - also pattern can be evaluated by looking at the residuals from a model without the predictor of interest

Evaluating individual observations

Outliers (lecture L1a)

- Large residuals
- Standardized residuals
  - expect 5% > 2 and < -2
  - expect 0.3% > 3 and < -3
- Deletion residuals
  - t-test - prob = 2*n*t(DFE-1, di)
- WPC log-transformed model – residuals
  - standardized residuals
    - expected 5% = 79 (observed = 50)
    - expected 0.3% = 5 (observed = 6)
    - expected and observed values very similar
  - deletion residuals
    - outlier cutoff value = 4.17
    - two observations >= this cutoff point
      - indication that these are outliers
        - cows with wpc = 1
    - delete and refit the model (later)
      - only for diagnostics purposes

Leverage (h)

- X-outliers
  - depend on X values only
    - not affected by relationship with y
  - expresses whether xi is outlying in distribution of x's
    - so potentially influential
  - less meaningful for categorical predictors
    - depends on the distribution of each category

- Cut-off values
  - concerns if hi ≥ 2(k+1)/n
    - or ≥ 3(k+1)/n
    - k=# of predictors, n=# obs.
    - should also look for unusual observations independently of these cutpoints
- Stata – predict varname, leverage
  - eg. predict lev_lnwpc, lev
  - lists/graphs to assess values

Computed after full model (ln_wpc)

```
. summm lev

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         lev |      1574    .0063532    .0083435   .0016636   .0642237

. display "leverage cutoff:  " 2*nparam/nobs
leverage cutoff:  .01270648

. display "conservative leverage cutoff:  " 3*nparam/nobs
conservative leverage cutoff:  .01905972

. count if lev>=.01905972 /* many (n=108) high leverage values */

. sort lev
. list wpc wpc_ln fit aut_calv herd_size parity1 twin dyst rp vag_disch stdres lev
in -10/-1
```

wpc	wpc_ln	fit	aut_calv	herd_s~e	parity1	twin	dyst	rp	vag_di~h	stdres	lev
40	3.689	4.407	yes	263	0	no	yes	yes	yes	-1.005	0.046
23	3.135	4.121	yes	185	0	yes	yes	no	no	-1.381	0.050
99	4.595	4.592	no	294	1	yes	yes	no	no	0.004	0.051
32	3.466	4.262	yes	201	1	yes	yes	yes	no	-1.117	0.052
110	4.700	4.162	yes	263	0	yes	no	no	yes	0.758	0.058
53	3.970	4.227	yes	263	5	yes	no	no	yes	-0.362	0.059
94	4.543	4.865	no	263	3	yes	no	yes	yes	-0.454	0.063
137	4.920	4.994	no	294	2	yes	no	yes	yes	-0.105	0.064
45	3.807	4.577	yes	201	4	yes	no	yes	yes	-1.089	0.064
76	4.331	4.699	no	185	4	yes	no	yes	yes	-0.520	0.064

```
. table twin rp dyst

-------------------------------------
          | Dystocia at calving and
          |   Retained placenta at
          |         calving
Twins     | --- no ---     --- yes --
born      |   no   yes       no   yes
----------+--------------------------
       no | 1332   124       76    15
      yes |   15     9        2     1
-------------------------------------
```

- large leverage values for less common combination of categorical predictors

Influence diagnostics: Cook's distance and DFITS

- Cook's distance and DFITS

Cooks D

$$D_i = \frac{r_{si}^2}{(k+1)} * \frac{h_i}{(1-h_i)}$$

DFITS

$$DFITS_i = r_{ti}\sqrt{\frac{h_i}{(1-h_i)}}$$

  - measure influence of an observation and have two interpretations:
    - effect of deleting observation "i" on the predictions
    - effect of outlier information about x's and y's
  - D_i is on the squared residual scale and based on standardized residuals
  - $DFITS_i$ is on absolute (signed) residual scale and are based on deletion residuals
- cut-off values
  - concern if either is >1 (rare) or if...
    - $D_i \geq 1$ or $\geq 4/n$
    - $DFITS_i$ outside $\pm 2*\sqrt{\frac{k+1}{n}}$ (if $n \geq 120$)
      - n < 120 only values beyond +/- 1
- Stata: predict varname, cooksd/dfits
  - lists/graphs to assess values

- Model: ln_wpc
  - Cook's

```
. summm cook

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        cook |      1574    .0006351    .0013679   5.57e-10   .0174304
```

```
.display "Cook's D cutoff:   " 4/nobs
   Cook's D cutoff:   .0025413
```

```
. count if cook>=.0025413 /* many (n=95) high Cook's D values */
  95
```

  - DFITS

```
. summ dfit

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        dfit |      1574    .0003931    .0797594  -.4179343   .3664328
```

```
. display "DFITS cutoff:   " 2*sqrt(nparam/nobs)*(nobs>=120)+1*(nobs<120)
     DFITS cutoff:   .15941443
```

```
. count if abs(dfit)>=.15941443 /* many (n=95) high DFITS values */
  95
```

  - largest Cook's values
    - not clear pattern - cows with at least one disease
  - largest negative DFIT values
    - cows with twins and at least one of the diseases were the most influential
  - largest positive DFIT values
    - cows without twins but with at least one of the diseases were the most influential

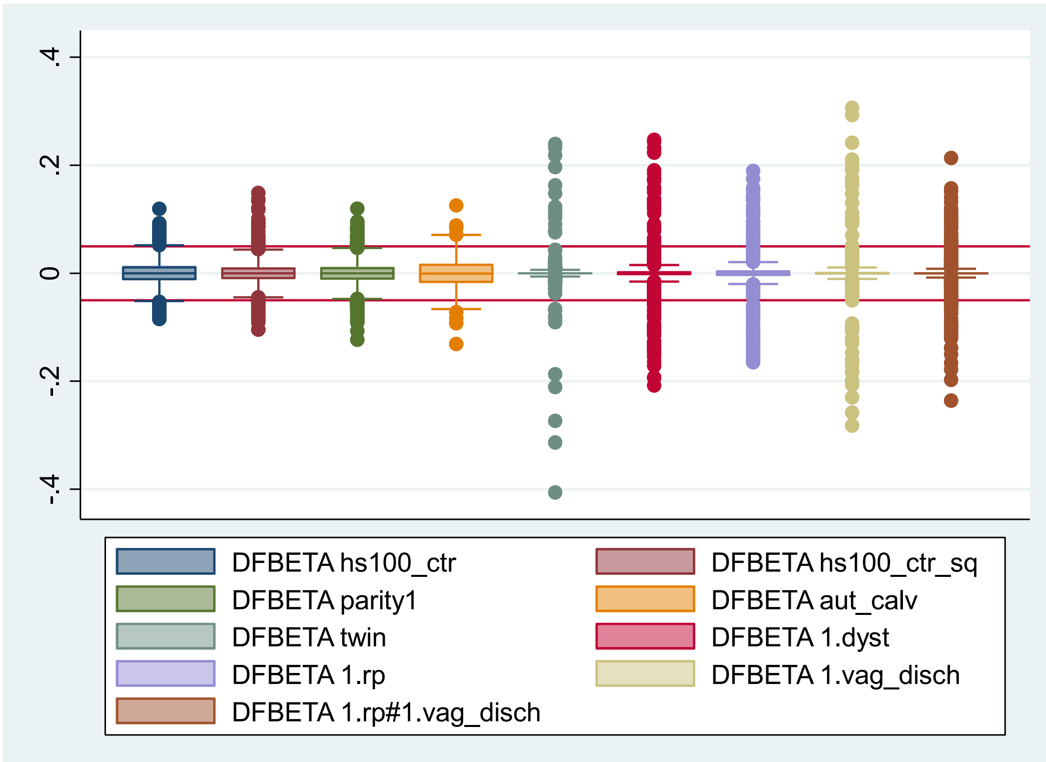
Delta-beta (or DFBETA)

- influence of an observation on a specific regression coefficient (β)
- for obs. "i" and predictor xj
  - $\text{DFBETA}_{j(i)} = \dfrac{(\beta_j - \beta_{j(i)})}{SE_j}$
  - where $\beta_{j(i)}$ is the estimate of β_j without obs. "i"
- measures change in β_j en terms of SE's
  - how many SE's does β_j increase(decrease) if "i" is dropped?

- cut-off values
  - $\text{DFBETA}_{j(i)} > 0$ (<0) ~ obs. "i" pulls β_j up (down)
    - extreme values of $\text{DFBETA}_{j(i)}$ are notable
    - most meaningful for continuous predictors
  - concern if outside $\pm\dfrac{2}{\sqrt{n}}$ (for n ≥ 120)
    - n < 120 only values beyond +/- 1
- Stata
  - predict varname, dfbeta(var in model)
    - eg. predict dfbdyst, dfbeta(dyst)
  - lists/graphs to assess values

- Model: ln_wpc

```
. display "DFBETA cutoff:   " 2/sqrt(nobs)*(nobs>=120)+1*(nobs<120)
DFBETA cutoff:   .05041127
```



- Explore values by browsing/listing
  - large values are cases of the correspondent predictor (eg. cows with diseases and twins)
  - linear effect of herd size similar to quadratic term
    - need to be aware of collinearity

What to do with Outliers/Influential observations

- Verify that point(s) (observation(s)) are not errors (eg. data entry errors)
- May be very informative
  - always examine observations and values of the predictors
- Determine if outliers are significant (L1a)
- Fit models with and without the outlier/influential observation(s)

- Keeping them in the analysis
  - potential for biased estimates
  - usually leads to larger SE and reduced power (is therefore conservative)
  - always report them
- Eliminating them from the analysis
  - should be always reported and justified
  - narrows scope of inference from the study
  - may create an unrealistically good model

Other transformations

- See "l2b_linear_reg_diag_other_transf.do"
- log-transformed model
  - some indication of non-constant variance
  - residuals still not normally-distributed
    - two severe outliers

```
. l wpc wpc_ln  fit herd_size parity dyst rp vag_disch if wpc==1, clean compress
noobs

    wpc   wpc~n        fit   her~e   par~y   dyst   rp   vag~h
      1       0   3.645548     201       2     no   no      no
      1       0   3.945861     263       2     no   no      no

. l wpc wpc_ln  delres lev dfit cook dfb_dyst dfb_rp dfb_vd dfb_int dfb_hs if wpc==1, clean
compress noobs

    wpc   wpc~n   delres     lev     dfit    cook   df~st   dfb~p   dfb~d   dfb~nt   dfb_hs
      1       0   -5.028   0.002   -0.243   0.006   0.051   0.037   0.028   -0.021    0.098
      1       0   -5.449   0.002   -0.243   0.006   0.044   0.053   0.042   -0.026   -0.016
```

  - comparison without outliers

```
. estimate table ln ln_noout
----------------------------------------
    Variable |     ln        ln_noout
-------------+--------------------------
   hs100_ctr |  .34365799    .33916323
hs100_ctr_sq |  .21187276    .20269649
     parity1 |  .01298436     .0113333
    aut_calv | -.13716214   -.13695147
        twin |  .39274261     .3894441
        dyst |   .1109405    .10338895
          rp |
        yes  |  .11231661    .10603408
   vag_disch |
        yes  | -.02601346   -.03328151
rp#vag_disch |
    yes#yes  |  .41369992    .42230092
       _cons |  3.8885867    3.9010244
----------------------------------------
```

  - repeat but without influential obs.
  - advantage log-transformed model
    - interpretation of estimates still possible

- Square root transformation
  - example VER – constant variance -> ok
  - residuals still not normally-distributed
  - deletion residuals show no indication of outlying observations
  - interpretation estimates not possible
    - obtain predicted values
    - parity=3 aut_calv=1 hsize=0 hsize2=0 twin=0

	Original scale			Sqrt transformation			log-transformation		
rp#vag_disch#dyst	est	lo_ci	up_ci	est	lo_ci	up_ci	est	lo_ci	up_ci
no#no#no	**66.605**	62.220	70.990	**58.267**	54.583	62.071	**50.127**	47.005	53.456
no#no#yes	**78.305**	67.338	89.272	**66.840**	57.187	77.244	**56.008**	47.688	65.780
no#yes#no	**67.833**	53.767	81.899	**58.064**	46.695	70.671	**48.840**	39.737	60.027
no#yes#yes	**79.533**	63.311	95.755	**66.622**	52.636	82.255	**54.570**	43.017	69.225
yes#no#no	**72.592**	63.048	82.135	**64.364**	56.085	73.213	**56.085**	48.761	64.510
yes#no#yes	**84.292**	70.918	97.666	**73.359**	61.107	86.731	**62.666**	51.506	76.243
yes#yes#no	**96.672**	78.396	114.947	**90.259**	71.883	110.723	**82.645**	63.217	108.045
yes#yes#yes	**108.372**	87.603	129.141	**100.857**	78.877	125.534	**92.342**	68.098	125.216

*ci estimated from t-distribution (Stata uses a different estimation procedure)

  - sqrt model – better than log-transformed
    - no outliers (eg. deletion res. range: -2.42; 3.28)
  - influential diagnostics
    - similar to log-transformed model
    - more in VER.