Index of Lecture 12–L

Page	Title
1	Transition models as Markov models
2	Transitional logistic regression
3	Indonesian children study
4	Transitional Poisson regression

— in addition to overheads from lectures 8–L and 11–L

Transition models as Markov models

Markov models:

- Markov property: "the future depends on the past only through the present",
- conditional — by successive conditioning on the past, e.g.,
  $P(Y_1, Y_2, Y_3, Y_4) = P(Y_1) \cdot P(Y_2|Y_1) \cdot P(Y_3|Y_1, Y_2) \cdot P(Y_4|Y_1, Y_2, Y_3)$,
- components of model:
  - initial distribution(s) — unconditional,
  - conditional distribution given past,
- order of model: number of time lags involved; simplest is first order where only previous observation involved,
- in general, no simple link between conditional and marginal models $\Rightarrow$ parameters have different interpretations.

Transitional logistic regression

Analysis of transitional logistic regression models:

- use previous state(s) (depending on order) as predictors along with other predictors,
- omit time point(s), where no previous state(s) available, or include them by *(i)* setting previous state(s) $\equiv 0$, and *(ii)* including indicator variable for obs. with no previous state(s) and its interactions,
- statistical analysis:
  - 2-level structure: ordinary logistic regression with robust variance estimation (clustered on subjects)[1] $\sim$ GEE with independence working correlation,
  - other hierarchical structures: marginal or random effects logistic regression.
- recommended to check sensitivity of results to order of model.

[1] Robust variance estimates gives consistent inference when (only) the conditional mean is correctly specified, and often also when the Markov property is violated.

Indonesian Children Study

Notes on models fitted in Diggle et al. (2002):

- first and second order transitional models,
- observations without previous states omitted,
- results for first order models:
  - previous state: a weak predictor,
  - estimates for Xerophthalmia only little affected by previous state,
  - no evidence of interaction with previous state, for Xerophthalmia or other predictors,
  - robust and ordinary SE's similar $\Rightarrow$ Markov assumption seems approximately ok,
- results for second order models:
  (not shown in tables, and I could not reproduce them from the dataset)
  - substantial increase in coefficient for Xerophthalmia,
  - reduced influence of season,

  — shows importance of assessing the robustness of results to model order.

Transitional Poisson regression

3 different models/approaches (for Poisson regression with log link):

- use Y_{t-1} as a predictor:

$$\log(\mu_{ti}) = (X'\beta)_{ti} + \gamma\, Y_{t-1,i},$$

  — does not work well for $\gamma > 0$ (exponential growth over time, rarely a realistic model), so primarily a model for negative correlation,

- use mean-corrected Y_{t-1} a a predictor:

$$\log(\mu_{ti}) = (X'\beta)_{ti} + \gamma\,(\log(Y^*_{t-1,i}) - (X'\beta)_{t-1,i}),$$

  where $Y^*_{t-1,i} = \max(d, Y_{t-1,i})$ for some $0 < d < 1$ (to avoid problems with zero counts);

  - $\gamma > 0 \sim$ positive correlation is possible and does not lead to non-stationarity,
  - model cannot be fit with standard GLM software,

- autocorrelated errors $\sim$ transitional model but usually considered as a GLM with autocorrelation/time series errors (Davis et al., Biometrika **87**:491–505),

- transitional regression models (Brumback et al., 2000) incorporate all above models (autocorrelated errors and transitional GLMs), and are fitted by generalized least squares (S code available from Statlib).