Henrik Stryhn

Index of Lecture 4–L

— in addition to overheads by Davis (2002):
76–79,89–95,97,108,111–13

Analysis of Multivariate Linear Model

Model: $Y = XB + E$:

- parameters B and Σ estimated by
  - $\hat{B} = (X'X)^{-1}X'Y \sim$ least squares estimate,
  - $\hat{\Sigma} = S = (Y - X\hat{B})'(Y - X\hat{B})/(n-p)$ unbiased estimate,
- usual ANOVA methods for single time point parameters $\beta_{1j}, \ldots, \beta_{pj}$,
- hypothesis of general form $\mathrm{H}_0 : ABC = D$, where
  - A is $(a \times p)$ matrix of coef. for "within time" hypotheses, and $\mathrm{rank}(A) = a \leq p$,
  - C is $(t \times c)$ matrix of coef. for "between time" hypotheses, and $\mathrm{rank}(C) = cp$,
  - D is $(a \times c)$ matrix of constants (often zeros),
- test of H_0:
  - 4 different test statistics: Wilk's lambda (likelihood ratio test), Pillai's trace, Hotelling-Lawley trace, and Roy's root statistic,
  - "one-dimensional" cases: agreement w. Hotelling's T^2,
  - except in simple cases, their distribution is known only approximately (by suitable F-distributions, usually indicated in the software),
  - no test generally "best" (power/robustness), but Wilk's lambda performs reasonably well in all situations.

Profile Analysis of Dental Data

- test of parallelism: previous lecture, $P = 0.070$,
- test of no group differences — two versions:
  * without assuming parallelism: previous lecture, $P = 0.020$,
  * assuming parallelism: univariate t-test between two groups (assuming same variances; similar without assumption),

$$t = 3.05, \quad \text{df} = 25, \quad P = 0.005,$$

- test of no time differences — two versions:
  * without assuming parallelism: Wilk's lambda converted to

$$F = 11.46, \quad \text{df} = (6, 46), \quad P < 0.0001,$$

  * assuming parallelism: Wilk's lambda or T^2-test,

$$F = 36.4, \quad \text{df} = (3, 23), \quad P < 0.0001,$$

- separate tests of no time differences:

$$\begin{aligned} \text{boys}: &\; F = 31.9, \quad \text{df} = (3, 23), \quad P < 0.0001, \\ \text{girls}: &\; F = 7.09, \quad \text{df} = (3, 23), \quad P = 0.0015. \end{aligned}$$

Conclusion:

- some indication of non-parallel curves,
- clear group and time differences.

Transformation Approach for Growth Curves

- idea: transform model to profile model,
- transformation $(t \times t)$ matrix G: either non-stochastic or independent of Y, so that $TG^{-1}T'$ has full rank (q), plus some other conditions,
- (Potthoff-Roy) procedure: computing

  $$Z = YG^{-1}T'(TG^{-1}T')^{-1},$$

  then Z is a $(n \times q)$ transformed data matrix for which the profile model $Z = XB + E^*$ holds.

Choice of T and polynomial order:

- full $(q = t)$ or high order for significance testing of coef. and to avoid loss of precision,
- low order to obtain easily interpretable model,
- calculations simplify for orthogonal polynomials but interpretation easier for polynomials in "natural" form.

Choice of G:

- if $q = t$ (full model): $Z = YT^{-1}$ independent of G,
- if $q < t$ (reduced model):
  - simplest choice is $G = I_t$ ($t \times t$ identity matrix) but possibly not most efficient,
  - better choice is $G = S$ for which the least-squares estimate of B based on Z is the ML estimate.

Growth Curve Analysis of Ramus Bone Data

Assessment of linearity using orthogonal polynomials:

Polyn. order	Estimate (95%CI)	t-test	P (from t_{19})
0: intercept	100.2 (97.8,102.5)	–	–
1: linear	2.12 (1.50,2.74)	7.20	$<$0.0001
2: quadratic	-0.02 (-0.24,0.20)	-0.19	0.85
3: cubic	0.002 (-0.21,0.21)	0.02	0.98

- univariate analyses of each component,
- combined test for quadratic and cubic in 3–L (non-sign).

Linear growth curve model:

$$y_{ij} = \beta_0 + \beta_1 \texttt{age}_j + \varepsilon_{ij},$$

analysed by transformation approach with $G = S$:

Term	Estimate (95%CI)	t-test	P (from t_{19})
intercept	33.39 (29.25,37.53)	–	–
slope	1.906 (1.440,2.373)	8.55	$<$0.0001

- estimates now on "real" scale (rather than orthogonalised), but with same strong significance,
- prediction equation: $\hat{y} = 33.39 + 1.906 \times \texttt{age}$,
- results using $G = I_4$: similar estimates, larger CI's.

Growth Curve Analysis for Dental Study

Assessment of linearity using orthogonal polynomials:

Polyn. order	Estimates (boys/girls)	F-test	P (from $F_{2,25}$)
0: intercept	49.4 / 45.3	–	–
1: linear	3.51 / 2.14	52.3	<0.0001
2: quadratic	0.41 / -0.02	1.27	0.30
3: cubic	-0.25 / 0.07	0.21	0.81

- F-tests: for hypotheses that polynomial term is negligible for both boys and girls,
- combined test of quadratic and cubic terms by Wilk's lambda: $F = 0.68$, $P = 0.61$ (from $F_{4,48}$).

Linear growth curve model:

$y_{hij} = \beta_{h0} + \beta_{h1}(\texttt{age}_j - 8) + \varepsilon_{hij}, \quad h = 1, 2 \sim$ (boys, girls),

analysed by transformation approach with $G = S$:

	Estimate (SE)			
Term	boys	girls	t-test	P (from t_{25})
int. (age=8)	22.46 (.51)	21.24 (.61)	1.53	0.14
slope	0.827 (.082)	0.476 (.099)	2.72	0.012

- estimates now on "real" scale, but age centred at 8 years,
- prediction equations:

$$\text{boys}: \ \hat{y} = 22.46 + 0.827 \times (\texttt{age} - 8),$$
$$\text{girls}: \ \hat{y} = 21.24 + 0.476 \times (\texttt{age} - 8).$$