Biostatistics VHM 882 Course
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Additional Exercise 2.2

This exercise deals with sample size and power calculation in basic normal distribution models.

Question 1. One-sample power calculation for known σ.

Assume $y_1, \ldots, y_n$ to be i.i.d. and $\sim \mathrm{N}(\mu, \sigma^2)$ with $\sigma > 0$ known. Compute the power for a z-test at significance level α of $\mathrm{H}_0 : \mu = 0$ against $\mathrm{H}_a : \mu > 0$, and give a formula for the required sample size n_β to achieve a desired power β.

Question 2. Two-sample power calculation for known σ.

Assume two i.i.d. and independent samples $y_1^{(1)}, \ldots, y_n^{(1)} \sim \mathrm{N}(\mu_1, \sigma^2)$ and $y_1^{(2)}, \ldots, y_n^{(2)} \sim \mathrm{N}(\mu_2, \sigma^2)$ with $\sigma > 0$ known. Compute in a similar manner as in **1.** the power for a z-test at level α of $\mathrm{H}_0 : \mu_1 = \mu_2$ against $\mathrm{H}_a : \mu_1 > \mu_2$, and show that the formula for the required sample size n_β to achieve a desired power β is the one on p. 42 in VER.

Question 3. One- and two-sample power calculations for unknown σ.

For $x \sim \mathrm{N}(\tau, 1)$ and $v \sim \chi^2(k)$, where x and v are independent variables, we define the non-central t-distribution with k degrees of freedom and non-centrality parameter τ as the distribution of

$$t' = \frac{x}{\sqrt{d/k}} \sim \text{ non-central } t(k, \tau).$$

Give formulae, in terms of the non-central t-distribution, for the power for the one- and two-sample situations discussed above when σ is an unknown parameter that needs to be estimated from the data. Use the formula and the one of **2.** to compute exact and approximate powers for the first setting in Additional Exercise 2.1; use R software to compute the non-central t-distribution percentile.

Question 4. Repeated measures formulae of the textbook.

Explain the derivation of the formulae 2.4.1 and 2.4.2 in the textbook (DH), and recompute the entries of the tables in Section 2.4.1 for $\rho = 0.8$ when the exchangeable (compound symmetry) correlation structure is replaced by a first-order autoregressive correlation structure.